MATH 2D Prep: Differentiation Techniques

Facts to Know:

1. Chain Rule: Two notations:
   (a) $\frac{d}{dx}[f(g(x))] =$ $f'(g(x)) \cdot g'(x)$
   (b) If $y =$ $f(u)$,$u =$ $g(x)$, then $\frac{dy}{dx} =$ $\frac{dy}{du} \cdot \frac{du}{dx}$
2. Product Rule: $\frac{d}{dx}[f(x)g(x)] =$ $f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Examples:

notation : $\sin x^2 = \sin(x^2)$

$\sin^2 x = [\sin x]^2$

1. Use the chain rule to find the derivative of $F(x) = \sin(x^2)$
   - Method 1: use notation same as in (a):

   $f(u) =$ $\sin(u)$ $g(x) =$ x^2 , $f'(u) =$ $\cos(u)$ $g'(x) =$ $2x$,

   $F'(x) = f'(g(x)) \cdot g'(x) = f'(x^2) \cdot 2x = \cos(x^2) \cdot 2x$

   - Method 2: use notation same as in (b):

   $y =$ $\sin(u)$ $u =$ x^2 , $\frac{dy}{du} =$ $\cos(u)$ $\frac{du}{dx} =$ $2x$,

   $F'(x) = \frac{dy}{dx} = \cos(u) \cdot 2x = \cos(x^2) \cdot 2x$

2. Find the derivative of $G(x) = x^3 \sin(x^2)$

   $f(x) =$ x^3 $g(x) =$ $\sin(x^2)$, $f'(x) =$ $3x^2$ $g'(x) =$ $2x\cos(x^2)$

   $G'(x) = f'g + fg' = 3x^2 \sin(x^2) + x^3 \cdot 2x\cos(x^2)$

   $= 3x^2 \sin(x^2) + 2x^4 \cos(x^2)$